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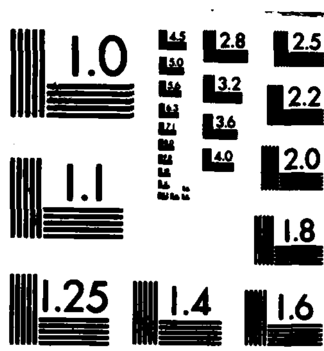

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A CRITIQUE OF DEMING'S DISCUSSION
OF ACCEPTANCE SAMPLING PROCEDURES

by

Richard E. Barlow and Xiang Zhang†

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DEDICATION

This paper is dedicated to W. Edwards Deming in honor of his 84th birthday.

ABSTRACT

Pi

An inspection sampling problem discussed by Deming ~~(1982) [see also Juran and Gryna (1980), pp. 336-337]~~ is analyzed assuming the percent defective, p , is partially unknown with prior $\pi(\cdot)$. In this case, the *all* or *none* inspection rule discussed by Deming may not be correct. Simple conditions based on the lot size, the ratio of costs, and the posterior mean determine when 100% inspection is best. Efficient algorithms based on the posterior mean are given for determining the optimal inspection plan in general. → cont keywords include: see 1473

A CRITIQUE OF DEMING'S DISCUSSION OF ACCEPTANCE SAMPLING PROCEDURES

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Richard E. Barlow and Xiang Zhang

1. INTRODUCTION

The basic thesis of this paper is that W. Edwards Deming is, at heart, a Bayesian. His interest in information and how to use it, together with his emphasis on solving real problems by minimizing expected costs, more than supports this thesis.

Deming (1982) Chapter 13, considers the following production situation. Periodically, lots of size N of similar units arrive and are put into assemblies in a production line. The decision problem is whether or not to inspect units before they are put into assemblies. If we opt for inspection, what sample size, n, of the lot size, N, should be inspected? Given x observed defectives in a sample of size n, the only decisions which will be considered are: $d_o(x,n)$ - install the remaining $N - n$ without inspection or $d_1(x,n)$ - inspect all of the remaining $N - n$ before installation.

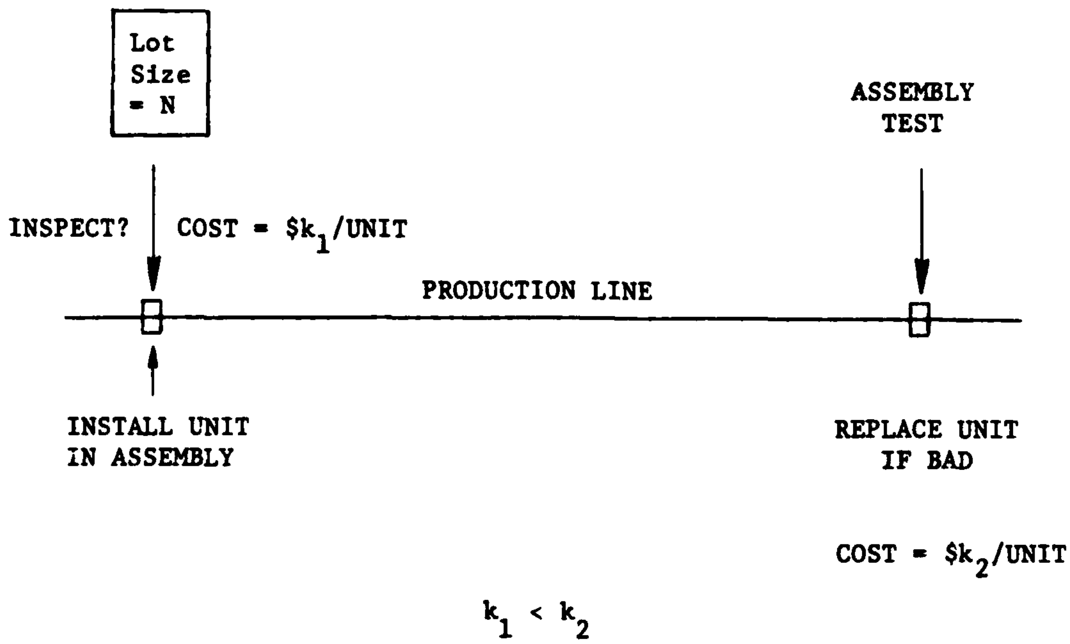
Let p be the percent defective over many lots obtained from the same vendor. Suppose we believe that the vendor's production of units is in *statistical control.* Let $\pi(p)$ be our probability assessment for this parameter p based on previous experience. $\pi(p)$ could be degenerate at, say p_o. In any event, haphazard sampling to check on the proportion defective in particular lots is prudent to update information on p.

Systematic lot sampling plans in Dodge-Romig (1929) and also implemented in Military Standard 105D (1963) are described in every textbook on quality control (e.g., Grant and Leavenworth (1974)). These have had the approval

of some government agencies and are widely used. *Deming (1982) argues that these sampling plans tend to maximize expected costs and should not be used.* The objective of these sampling plans is to provide a procedure whereby a decision as to whether or not to reject the lot can be made automatically. If the lot is rejected, it could be sent back to the vendor or it could be subjected to 100% inspection with defective units replaced by good units. Deming (1982) argues that the vendor should supply quality records to the manufacturer and that they should work together on the quality problem as opposed to simply sending bad lots back to the vendor. If lots determined to be bad, based on a sample, are to be subjected to 100% inspection and bad units replaced by good units at vendor's expense, what should the sample size, n, be? *Deming argues that, given prior information on* p, *the minimum cost choice for* n *is either all or none; i.e.,* $n = N$ *or* $n = 0$. This choice is determined by the cost of sampling, the cost of replacing a defective unit in the assembly by a good unit and $\pi(p)$. We will determine conditions when this is correct and conditions when it is not.

(Deming (1982) points out on page 270 that there may be conditions under which the all or none rule is not optimal but he does not furnish explicit conditions.) Figure 1 describes our production setup.

Juran and Gryna (1980, section 17.2) consider the same problem when p is known and they credit the solution to Martin (1964). The mathematics of sampling inspection plans are discussed by Hald (1981). However, we follow the notation and mathematical development of Deming (1982). Let k_1 be the expected cost to inspect one unit at the start. Let k_2 be the cost of a defective unit that gets into the production-line. This cost will include the cost to tear down the assembly at the point where the defect is discovered. In some instances, we must replace the unit and retest the assembly. Let Z_i be 1 if the i-th unit is defective and 0



DEMING'S INSPECTION PROBLEM

FIGURE 1

otherwise. Let $CI(Z_i)$ be the cost of inspecting unit i. If a unit is inspected and found defective, additional units *from another lot* are inspected until a good unit is found. (We make this model assumption since all defective units which are found will be replaced at vendor's expense.) Hence, conditional on p, the expected cost of inspecting unit i and, if the unit is bad, inspecting until a good unit is found is

$$E[CI(Z_i) \mid p] = k_1[1 + p/q]$$

where $q = 1 - p$.

Let $CR(Z_i)$ be the cost of replacing unit i, if required, when it is *not* inspected. Conditional on p, the expected cost of replacing unit i in the assembly is

$$E[CR(Z_i) \mid p] = p[k_2 + k_1/q] .$$

Note that, since the replacement *must* be good, there is again an inspection cost incurred to find a good unit.

The usual sampling plan depends on n and a critical number c. If the observed number of defectives in the sample is less than or equal to c, the lot is accepted and otherwise rejected. If the lot is accepted, no further inspection is made. If the lot is rejected, the remaining $N - n$ are inspected. Let $P(rej \mid n,c,p)$ be the prior conditional probability that, under this inspection plan, all N units will be inspected. Let $P(acc \mid n,c,p) = 1 - P(rej \mid n,c,p)$ be the corresponding prior conditional probability that the remaining $N - n$ will *not* be inspected.

Given p, the expected cost of inspection of a lot of N incoming parts will be

$$E\left[\sum_{i=1}^{n} CI(Z_i) \mid p\right] + P(rej \mid n,c,p)E\left[\sum_{i=n+1}^{N} CI(Z_i) \mid p\right]$$
$$= k_1[n + (N - n)P(rej \mid n,c,p)](1 + p/q) .$$

Given p , the expected cost from the defective units that get into the production line will be

$$P(acc \mid n,c,p)E\left[\sum_{n+1}^{N} CR(Z_i) \mid p\right]$$
$$= (N - n)P(acc \mid n,c,p)p[k_2 + k_1/q]$$

where $P(acc \mid n,c,p) = 1 - P(rej \mid n,c,p)$. Let $r = n/N$ and note that the total expected cost, given p , is

$$k_1N\{[r + (1 - r)P(rej \mid n,c,p)](1 + p/q)$$
$$+ (1 - r)P(acc \mid n,c,p)[pk_2/k_1 + p/q]\} .$$

Substituting $P(rej \mid n,c,p) = 1 - P(acc \mid n,c,p)$ we have

$$k_1N\{1 + p/q + (1 - r)[-P(acc \mid n,c,p)(1 + p/q)$$
$$+ P(acc \mid n,c,p)(pk_2/k_1 + p/q)]\}$$
$$= k_1N\{1 + p/q + (1 - r)P(acc \mid n,c,p)(pk_2/k_1 - 1)\} . \tag{1.1}$$

Since $P(acc \mid n,c,p) \geq 0$, we see that the conditional total expected cost is minimized for $r = 0$ or $n = 0$ given p , when

$$(pk_2/k_1 - 1) < 0 \quad \text{or} \quad p < k_1/k_2$$

and for $r = 1$ or $n = N$ when

$$(pk_2/k_1 - 1) \geq 0 \quad \text{or} \quad p \geq k_1/k_2 .$$

If our prior density $\pi(p) = 0$ for $p > k_1/k_2$, i.e., we feel certain that $p < k_1/k_2$, then $n = 0$ is optimal.

If, on the other hand, our prior density $\pi(p) = 0$ for $p < k_1/k_2$, i.e., we feel certain that $p > k_1/k_2$ then $n = N$ is optimal.

These results are recorded in Deming (1982) with no mention of a possible prior density $\pi(p)$. For this reason, the case when $\pi(p)$ contains k_1/k_2 in an interval of its support was not considered. As we shall see, the all or none results are not necessarily valid in this case.

2. CONDITIONS WHEN $0 < n < N$

The solution to a similar problem, when p is partially unknown with prior $\pi(p)$, was discussed by Hald (1960). However, he considered a hypergeometric rather than binomal model. His solutions are more complicated.

The purpose of sampling is to gain information about p. If we think we know p or if we think either $p < k_1/k_2$ or $p \geq k_1/k_2$ (but of course, not both), then Deming is right - we should not sample. However, if we are sufficiently uncertain about p, then our minimum expected cost strategy may be to sample. The following results establish when $0 < n < N$ and suggest an algorithm for determining the optimal (n,c) inspection policy. The Beta(A,B) density, $\pi(p) \prec p^{A-1}(1 - p)^{B-1}$ is often used in practice. However, our results are valid for an arbitrary prior, $\pi(p)$.

Theorem 1:

If $\pi(p \mid n,x)$ is the posterior density for p given x defectives in a sample of size n, then $n = N$ *is optimal* if and only if

$$\int_0^1 p\pi(p \mid N - 1 , x = 0)dp > k_1/k_2 . \qquad (2.1)$$

For the Beta(A,B) prior this means $n = N$ if and only if

$$A/(A + B + N - 1) > k_1/k_2 .$$

Theorem 2:

If (2.1) does not hold, then the optimal $n \geq n_o$ where

$$n_o = \max [0,n^*]$$

and n^* is the smallest value of n such that

$$\int_0^1 p\pi(p \mid n , x = 0)dp \leq k_1/k_2 . \qquad (2.2)$$

For the Beta(A,B) density

$$n_o = \max \{0, [(k_2/k_1)A - A - B]\} .$$

Theorem 3:

For n such that $n_o \le n \le N$, the optimum value of c is the largest value of c such that

$$\int_0^1 p\pi(p \mid n,c)dp \le k_1/k_2 .$$

For the Beta(A,B) density, the solution $c(n)$ is

$$c(n) = \min \{n, [(k_1/k_2)(n + A + B) - A]\} .$$

Theorem 4:

The policy $(n, c(n) = n)$ and $0 < n < N$ is *never* the only optimal solution.

It follows from Theorems 1 and 2 that if

$$\int_0^1 p\pi(p)dp > k_1/k_2$$

and

$$\int_0^1 p\pi(p \mid N - 1 , x = 0)dp < k_1/k_2$$

then the optimal n is neither 0 nor N.

These results suggest the following algorithm. If $k_1 \ge k_2$ then $n = 0$ is optimal. If

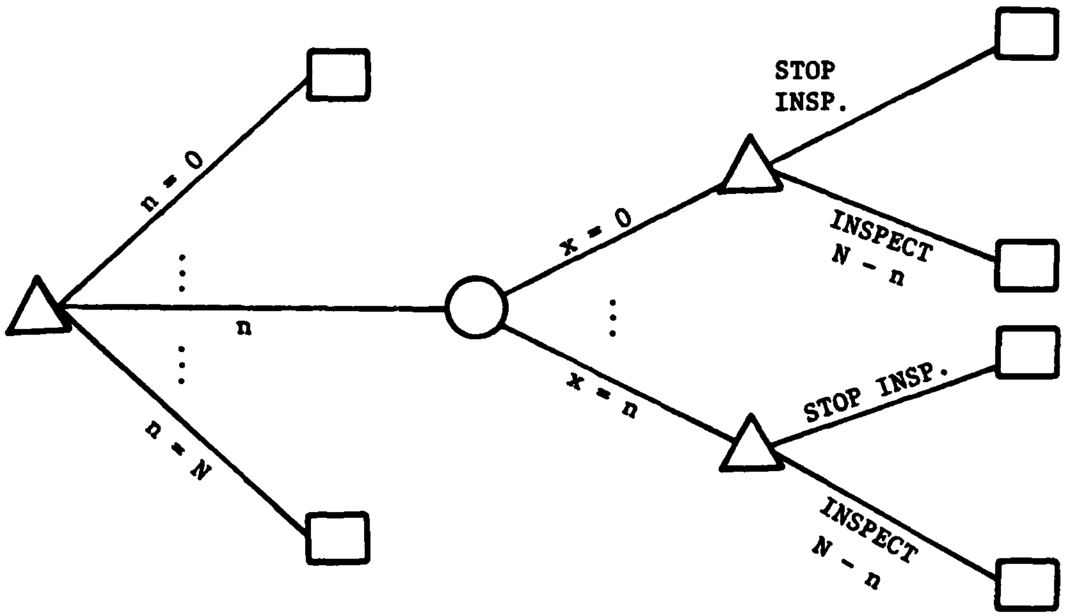
$$\int_0^1 p\pi(p \mid N-1, x=0)dp > k_1/k_2$$

then $n = N$, else compute n_o from Theorem 2. For each $n \geq n_o$, find the optimal $c(n)$ using Theorem 3 and record the expected cost. Select that $(n,c(n))$ policy having minimum expected cost. Another algorithm will be given in the Appendix.

If we stop inspection, we can clearly gain no further information about p and hence there would be no reason to resume inspection once stopped. However, if there is a feedback loop from assembly tests, then we may update our last posterior density for p and determine an optimal inspection policy for remainders.

3. PROOFS OF THEOREMS

Our decision problem can be described by a decision tree as in Figure 2.



DECISION TREE FOR INSPECTION PROBLEM

FIGURE 2

We will need the following lemma based on Figure 2.

Lemma 1.

Given (x,n) and restricting decisions to $d_o(x,n)$ and $d_1(x,n)$, $d_o(x,n)$ is best if

$$\int_0^1 p\pi(p \mid n,x)dp < k_1/k_2$$

and $d_1(x,n)$ otherwise.

Proof:

The expected cost given (x,n) if we take decision $d_o(x,n)$ is

$$C(d_o) = E_{\pi(\cdot)}[(N - n)p(k_2 + k_1/q) \mid n,x]$$
$$= (N - n)E_{\pi(\cdot)}[p(k_2 + k_1/q) \mid n,x] .$$

If decision $d_1(x,n)$ is taken

$$C(d_1) = (N - n)E_{\pi(\cdot)}[k_1(1 + p/q) \mid n,x] .$$

Hence $d_o(x,n)$ is optimal if $C(d_o) < C(d_1)$ or

$$\int_0^1 p\pi(p \mid n,x)dp < k_1/k_2 . \qquad \text{Q.E.D.}$$

By the principle of maximizing expected utility (or minimizing expected cost in this case) if n are sampled and x are found defective, then we should STOP inspection if

$$\int_0^1 p\pi(p \mid n,x)dp \leq k_1/k_2$$

and inspect the remaining $N - n$ otherwise in order to minimize our expected cost given n and x . This principle and the monotone likelihood ratio property, true for the binomial likelihood, are all that are needed to prove our theorems. Clearly, many similar theorems can be proved for other probability models which also have the monotone likelihood ratio property. In the binomial case, the likelihood

$$L(p \mid n,x) \prec p^x(1 - p)^{n-x}$$

has the property that if $p_1 < p_2$ then small values of x support p_1 while large values of x support p_2 in the likelihood ratio sense; i.e.,

$$\frac{L(p_1 \mid n,x_1)}{L(p_2 \mid n,x_1)} \geq \frac{L(p_1 \mid n,x_2)}{L(p_2 \mid n,x_2)} \tag{3.1}$$

if $x_1 \leq x_2$. The inequality is reversed if x is fixed, n_1 (n_2) replaces n in the left (right) ratio and $n_1 < n_2$.

The following lemma is well known and is similar to Lemma 1 in Karlin and Rubin (1956a).

Lemma 2:

Let $\pi(p)$ be a prior for p. Let (x,n) be data and suppose that for $p_1 < p_2$

$$L(p_1 \mid n,x)/L(p_2 \mid n,x)$$

is $\downarrow$ in x and $\uparrow$ in n. Let $g(p)$ be $\uparrow$ in p. Then

$$\int_0^1 g(p)\pi(p \mid n,x)dp$$

is $\uparrow$ in x (n fixed) and $\downarrow$ in n (x fixed).

Proof of Theorem 1:

Suppose we inspect $N - 1$ units and observe $x = 0$ defectives. Given this information, our optimal decision is to inspect the remaining unit by (2.1) and lemma 1. By (2.1) and lemma 2

$$\int_0^1 p\pi(p \mid n,x)dp > k_1/k_2 \ .$$

Hence, if (2.1) is true, then any inspection policy which is optimal after sampling will result in complete inspection. It follows under (2.1) that $n = N$ is optimal. Conversely, if $n = N$ is optimal, then (2.1) must be true. Q.E.D.

Proof of Theorem 2:

By Theorem 1, if (2.1) does not hold then $n = N$ is not optimal. Suppose n is optimal but does not satisfy (2.1) or (2.2). Then

$$\int_0^1 p\pi(p \mid n \ , \ x = 0)dp > k_1/k_2$$

which by lemma 2 implies

$$\int_0^1 p\pi(p \mid n,x)dp > k_1/k_2 \ ,$$

for all x , $0 \leq x \leq n$. This implies $n = N$ was optimal after all which is a contradiction. Hence $n \geq n_o$ as claimed. Q.E.D.

Proof of Theorem 3:

After inspecting n units, consider the best of the two decisions:

d_o : STOP further inspection

d_1 : INSPECT the remaining $N - n$.

Given n inspections and x observed defectives, d_o is best if

$$\int_0^1 p\pi(p \mid n,x)\,dp \le k_1/k_2$$

and d_1 otherwise. Hence the optimal $c(n)$ is the value stated, again by the lemma. Q.E.D.

<u>Proof of Theorem 4</u>:

By (1.1), we need only calculate

$$(1 - n/N)E_{\pi(\cdot)}\left\{P(\text{acc} \mid n,c,p)\left(p\,\frac{k_2}{k_1} - 1\right)\right\} \tag{3.2}$$

to determine the best policy.

For $(n,c(n))$, $P(\text{acc} \mid n,n,p) = 1$ so that (3.2) becomes

$$(1 - n/N)E_{\pi(\cdot)}\left\{p\,\frac{k_2}{k_1} - 1\right\} . \tag{3.3}$$

If $(n,c(n) = n)$ were better than $n = N$, then

$$E_{\pi(\cdot)}\left\{p\,\frac{k_2}{k_1} - 1\right\} < 0 . \tag{3.4}$$

But in this case, $n = 0$ would be even better. Hence either $n = 0$ or $n = N$ is better or just as good as $(n,c(n) = n)$. Q.E.D.

4. COMMENTS

In his book, Deming (1982) says, "Inspection of incoming materials is an economic problem, and should be so treated. None of the standard acceptance plans, so popular in courses and textbooks, minimizes the total cost ... Put another way, they minimize the wrong cost." More generally, Deming should have said that optimal decision rules can only be obtained by maximizing expected utility. In the special case when our utility for money is approximately linear in money, this can be done by minimizing our expected cost (calculated using a density measuring our uncertainty about unknown but relevant quantities). Our utility function need not be exclusively based on money, but it must be coherently assessed. For an excellent account of why this approach makes sense, read Lindley, *Making Decisions* (1971).

The simple decision problem considered here is a special case of two decision problems considered more generally in Karlin (1956b) but from a *non-Bayesian* point of view.

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APPENDIX

We present here an alternative algorithm for computing the optimal n and c. This algorithm, justifiable from the theorems stated in our paper, differs from the previous one in that, instead of finding the optimal $c(n)$ for each n and then comparing all the pairs $(n,c(n))$ for $n = 0,1,2, \ldots, N$, to get the one with minimum expected cost, it uses the same Theorem 3 in reverse to get the set of n, $S(c) = \{n : n_1(c) \leq n \leq n_2(c)\}$ for each c, such that $c(n)$ in Theorem 3 equals c for all n in the set. It then finds the locally optimal $(n(c),c)$ within $S(c)$ first, and then compares all the pairs $(n(c),c)$ for c running over its range of interest to obtain the optimal (n,c) plan.

In general, both algorithms have the same computational complexity. But for certain prior distributions, the Beta distribution for instance, the value of the objective function is unimodal in n belonging to the set $S(c)$. So, in this case, we can use a binary search (instead of a linear search) for $(n(c),c)$, and greatly reduce the computational effort.

The algorithm goes like this:

Step 0: If $k_1/k_2 \geq 1$, then $n = 0$ is optimal. Stop. Or else, if

$$\int_0^1 p\pi(p \mid N - 1 , x = 0)dp > k_1/k_2 ,$$

(for a Beta(A,B) prior, this inequality becomes $A/(A + B + N - 1) > k_1/k_2$) then $n = N$ is optimal. Stop. Or else, compute the range of c, $0 \leq c \leq c_o$, where c_o is the largest c such that $c_o \leq N$ and

$$\int_0^1 p\pi(p \mid N, x = c_o)dp \le k_1/k_2 .$$

For Beta(A,B) , $c_o = \min\{N, [(k_1/k_2)(A + B + N) - A]\}$.

Set $c = 0$. Go to Step 1.

Step 1: Find $S(c) = \{n : n_1(c) \le n \le n_2(c)\}$, where $n_1(c)$ is the smallest n such that $n \ge c$ and

$$\int_0^1 p\pi(p \mid n,c)dp \le k_1/k_2 ;$$

$n_2(c)$ is the smallest n such that $n \ge c$ and

$$\int_0^1 p\pi(p \mid n + 1,c + 1)dp \le k_1/k_2 .$$

For Beta(A,B) ,

$n_1(c) = \max\{c,[(k_2/k_1)(c + A) - A - B]\}$;

$n_2(c) = \max\{c,[(k_2/k_1)(c + A) - A - B + k_2/k_1] - 1\}$.

Go to Step 2.

Step 2: In $S(c)$, find $n(c)$ such that $(n(c),c)$ yields the local minimum in the objective function; record $n(c)$, c , and the local optimal value LOCopt(c) .

$c \leftarrow c + 1$. If $c \le c_o$, go to Step 1. Or else, go to Step 3.

Step 3: Compare LOCopt(c) for $c = 0,1, \ldots, c_o$ to obtain the (global) optimal (n,c) plan. Stop.

For the Beta prior, because of a binary search in Step 2, the total number of comparisons is, in the worst case,

$$\left(\frac{\log_2(k_2/k_1)}{k_2/k_1}\right) \cdot N .$$

For a numerical result, we use the data from the practical example on pp. 279-280, Chapter 13, Deming (1982).

Lot size $N = 2800$; inspection cost $k_1 = \$.07$; tear-down cost $k_2 = \$15$ so that $k_1/k_2 = 4.66 \times 10^{-3}$. p has a prior Beta(A,B) such that $E(p) = A/(A + B) = .01$, which is the value for p in Deming's example.

$A = .1$, $B = 9.9$ $\sqrt{Var\ (p)} = 0.03$

Inspection of incoming rods	Expected cost per lot
None	\$422
100%	\$198
(146,0)	\$ 77

$A = 1$, $B = 99$ $\sqrt{Var\ (p)} = 0.01$

Inspection of incoming rods	Expected cost per lot
None	\$422
100%	\$198
(630,2)	\$174

$A = 10$, $B = 990$ $\sqrt{Var\ (p)} = 0.003$

Inspection of incoming rods	Expected cost per lot
None	\$422
100%	\$198
(2098,4)	\$197.97

As we can see in the previous example, the all or none policy is not as good as an (n,c) plan for some $0 < n < N$, especially when there is sufficient uncertainty concerning p relative to k_1/k_2. We also see that with uncertainty reduced, the optimal (n,c) plan tends to approach either the all or the none policy.

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